EMSE314--Fall 2003--Homework 2--Name: Changrong Li
Issued: Sep. 15, Due: Sep. 24.

Problem 1 (20 pts)

(a) (2 pts) Even though P_1 and P_2 are in non-parallel directions, they are still equivalent because they are related by a 60° rotation around the ***c*** axis, the characteristic symmetry operation of the hexagonal system.

P_1 (-1 1 0)
P_2 (-1 0 0)
P_3 (-1 2 0)
P_4 (-1 -1 0)

(b) (2 pts) P_1 (-1 1 0 0)
P_2 (-1 0 1 0)
P_3 (-1 2 -1 0)
P_4 (-1 -1 2 0)

(c) (4 pts) area (ΔOAB) = area (ΔOAD) + area (ΔODB)

$$\frac{1}{2}\frac{a}{h}\frac{b}{k}\sin 120° = \frac{1}{2}\frac{a}{h}\frac{d}{-i}\sin 60° + \frac{1}{2}\frac{b}{k}\frac{d}{-i}\sin 60°$$

$a = b = d$ (Please note that $OD=d/(-i)$) (Eq. 1)

$$\frac{1}{hk} = -\frac{1}{hi} - \frac{1}{ki}$$

$h + k + i = 0$ (Eq. 2)

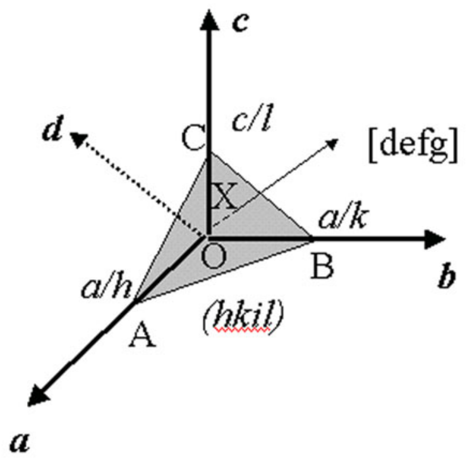
(d) (4 pts) $U\boldsymbol{a} + V\boldsymbol{b} + W\boldsymbol{c} = u\boldsymbol{a} + v\boldsymbol{b} + t\boldsymbol{d} + w\boldsymbol{c}$
$= u\boldsymbol{a} + v\boldsymbol{b} + t(-\boldsymbol{a}-\boldsymbol{b}) + w\boldsymbol{c}$
$= (u-t)\boldsymbol{a} + (v-t)\boldsymbol{b} + w\boldsymbol{c}$

$U = u - t = 2u+v$ $V = v - t = u+2v$ $W = w$ (2 pts) (Eq. 3)

$u + v + t = 0$ (Eq. 4)
$U = u-t$
$V = v-t$

$u = (2U-V)/3$ $v = (2V-U)/3$ $t=-(U+V)/3$ $w = W$ (2 pts) (Eq. 5)

(e) (8 pts)



As shown in the diagram, plane *(hkil)* intersects the axes at points A, B and C.

For a hexagonal system, we have $a = b = d, \vec{a} \bullet \vec{b} = -1/2, \vec{a} \bullet \vec{c} = \vec{b} \bullet \vec{c} = 0$ (1 pt)

where $\vec{a}$, $\vec{b}$, $\vec{c}$ and $\vec{d}$ are unit vectors along each axes.

By definition of *(hkil)* plane

$$\overrightarrow{OA} = \frac{a}{h}\vec{a},\ \overrightarrow{OB} = \frac{b}{k}\vec{b}, \overrightarrow{OC} = \frac{c}{l}\vec{c},$$

$$\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA} = \frac{b}{k}\vec{b} - \frac{a}{h}\vec{a},\ \overrightarrow{BC} = \overrightarrow{OC} - \overrightarrow{OB} = \frac{c}{l}\vec{c} - \frac{b}{k}\vec{b}, \overrightarrow{CA} = \frac{a}{h}\vec{a} - \frac{c}{l}\vec{c}.$$ (1 pt)

Say [defg] direction, the normal of *(hkil)* plane, intersect it at point X. By definition,

$$\overrightarrow{OX} = x(da\vec{a} + eb\vec{b} + fd\vec{d} + gc\vec{c}) = (2xda + xea)\vec{a} + (xda + 2xea)\vec{b} + xgc\vec{c}.$$

Here we adopt the fact that *f=-(d+e)*, result from question (d), and that a=b=d.

Vectors $\overrightarrow{OX}$ and $\overrightarrow{AB}$ are perpendicular to each other so that $\overrightarrow{OX} \bullet \overrightarrow{AB} = 0$.

$$0 = \overrightarrow{OX} \cdot \overrightarrow{AB} = -\frac{a}{2k}(2xda + xea) + \frac{b}{k}(xda + 2xea) - \frac{a}{h}(2xda + xea) + \frac{a}{2h}(xda + 2xea)$$

Rewrite it ($\times\frac{2hk}{3xa^2}$): $0 = he - kd$ (1.5 pts) (Eq. 6)

Similarly,

$$0 = \overrightarrow{OX} \cdot \overrightarrow{BC} = \frac{1}{l}xc^2 g + \frac{a}{2k}(2xda + xea) - \frac{a}{k}(xda + 2xea) = \frac{1}{l}xc^2 g - \frac{3xa^2}{2k}e$$

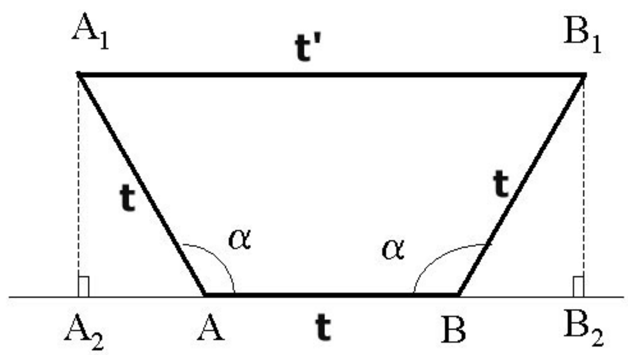
Rewrite it ($\times\frac{kl}{x}$): $0 = kc^2 g - \frac{3la^2}{2}e$ (1.5 pts) (Eq. 7)

From the definition of direction [defg], we can arbitrarily choose one of these indices, say let d=h. Substituting into equation 6, we can get e=k. Using equations 2 and 4,

$f = -(d + e) = -(h + k) = i$. Then from equation 7, $g = \frac{3la^2}{2kc^2}k = \frac{3a^2}{2c^2}l$. (3 pts)

In summary, $[defg] = h, k, i, \frac{3a^2}{2c^2}l$.

Problem 2 *(10 pts)*



Let's start with a row of lattice points. Consider two lattice points A and B separated by a unit translation *t*. If we rotate AB counterclockwise through an angle α as shown in the schematic diagram, we can get another lattice point A_1. If we rotate AB clockwise, we can get lattice point B_1. Lines A_1A_2 and B_1B_2 are perpendicular to line AB at points A_2 and B_2, respectively.

$$A_1A_2 = B_1B_2 = t\sin(\pi - \alpha) = t\sin\alpha$$

Therefore $A_1B_1 // A_2B_2$ (2 pts)

Moreover, A_1 and B_1 are also lattice points so that

$A_1B_1 = t' = mt$, where m is an integer. (2 pts) (eq. 1)

From the geometry, it is clear that $A_1A_2B_2B_1$ is a rectangle.

Hence $A_1B_1 = A_2B_2 = t + 2t\cos(\pi - \alpha) = t - 2t\cos\alpha$ (eq. 2)

Substituting Eq. 1 into Eq. 2, rewriting it, we can get that

$$\cos\alpha = \frac{1-m}{2}$$ (2 pts) (eq. 3)

Considering |cosα|<=1, (3 pts)

m	3	2	1	0	-1
cosα	-1	-1/2	0	1/2	1
α	π	2π/3	π/2	π/3	2π
n	2	3	4	6	1

It is worth discussing with what we have deduced. (1 pt)

If n=6 (i.e. α=π/3), point A_1 coincides with point B_1. In this case, $t' = 0 = 0t$ so that eq.1 and eq. 3 are also true.

If n=2 (i.e. α=π), line A_1B_1 is identical with A_2B_2, $t' = 3t$, eq.1 and eq. 3 are still true.

If n=1, we don't even need to prove anything because every crystal has 1-fold symmetry.

If n equals to other values, the above geometry and argument are true.

In conclusion, n must be 1, 2, 3, 4 or 6.

Another interesting solution is given as the following.
Any proper rotation through an angle α about the c-axis is given with respect to orthogonal Cartesian axes by the matrix

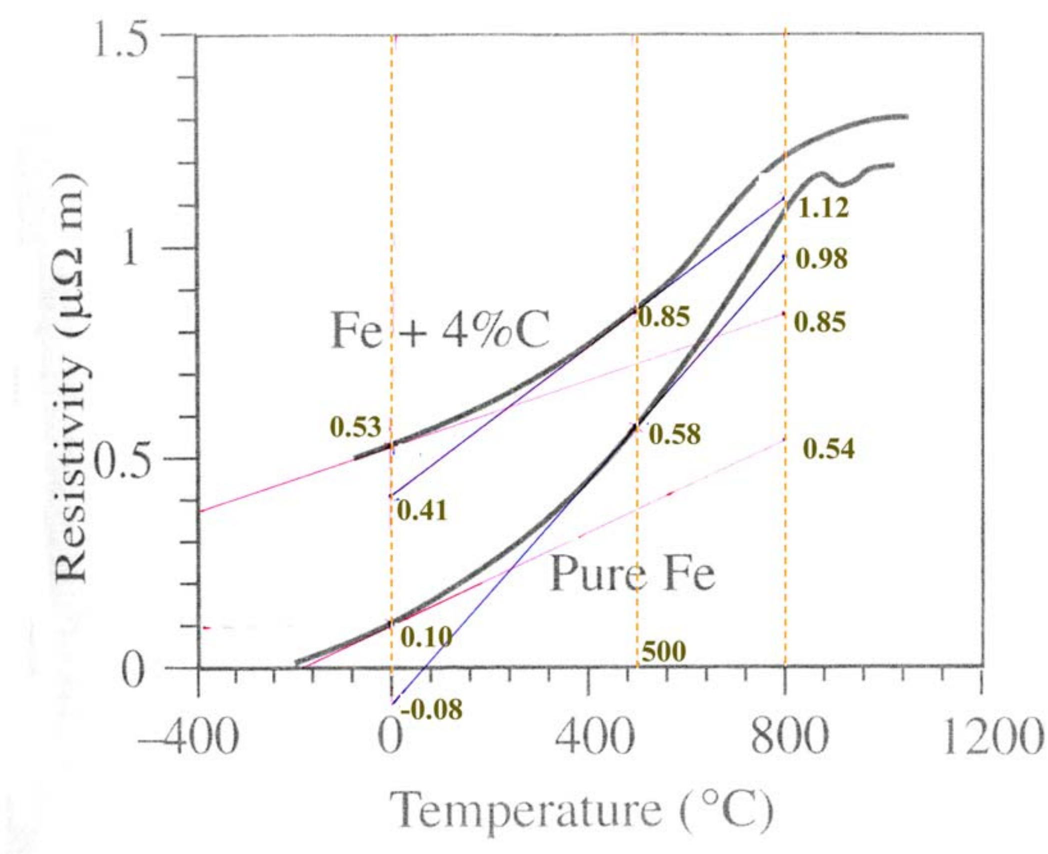
$$\begin{bmatrix} \cos\alpha & -\sin\alpha & 0 \\ \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

As is well known, the trace of such a matrix (sum of diagonal elements) must be equal to an integer when we are dealing with a symmetry operation.

$$1 + 2\cos\alpha = \text{integer} = 3,\ 2,\ 1,\ 0,\ \text{-}1.$$

Therefore, $\cos\alpha = 1, 1/2, 0, \text{-}1/2, 1$, which give n=1, 6, 4, 3 and 2, respectively. (10 pts)

Problem 3 *(10 pts)*
(a) Draw the slopes as the following (1 pt)



The temperature coefficient of resistivity at T_0 is defined by

$$\alpha_0 = \frac{1}{\rho_0}\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}$$, where ρ_0 is the resistivity at T_0. (1 pt)

Materials	Pure iron		Fe +4%C	
T_0	0	500	0	500
ρ_0 (μΩ.m)	0.10	0.58	0.53	0.85
$\left[\frac{\delta\rho}{\delta T}\right]_{T=T_0}$ (μΩ.m/K)	$\frac{0.54-0.10}{800} = 5.50\times10^{-4}$	$\frac{0.98+0.08}{800} = 1.33\times10^{-3}$	$\frac{0.85-0.53}{800} = 4.00\times10^{-4}$	$\frac{1.12-0.41}{800} = 8.88\times10^{-4}$
α_0 (1/K)	0.0055=1/182	0.0023=1/436	0.00075=1/1325	0.0010=1/958

(4 pts)

(b) According to Matthiessen's rule, $\rho = \rho_T + \rho_R$, where ρ_T is resistivity due to scattering from thermal vibrations, ρ_R is residual resistivity due to the scattering of electrons by crystal defects (such as impurities, dislocations...).
Steel contains interstitial carbon atoms so that it has higher resistivity than pure iron. Moreover, steel shows a residual resistivity while pure iron doesn't. (2 pts)
Carbon interstitials will hinder the lattice vibration in steel. As a result, temperature coefficient of resistivity of steel is less than that of pure iron. (2 pts)

Problem 4 (*10 pts*)
(a) Following the procedure in section 2.5 of the textbook. Note the directions of Lorentz force, carries movement, and current. (*6 pts*)

(b) Given $n = p = 1.8\times10^6 cm^{-3}$, $\mu_e \approx 8500cm^2/Vs$, and $\mu_h \approx 400cm^2/Vs$, we have $b = \mu_e/\mu_h = 8500/400 = 21.25$.
The Hall coefficient of intrinsic semiconductor is given as

$$R_H = \frac{p\mu_h^2 - n\mu_e^2}{e(p\mu_h + n\mu_e)^2} = \frac{p\text{-}nb^2}{e(p+nb)^2} = \frac{p-pb^2}{e(p+pb)^2} = \frac{1-b}{ep(1+b)},$$ where $b = \mu_e/\mu_h$

(*3 pts*)

$$\frac{1-21.25}{1.6\times10^{-19}\times1.8\times10^6\times10^6\times(1+21.25)} \text{m}^3\text{A}^{-1}\text{s}^{-1} = -3.16\times10^6\ \text{m}^3\text{A}^{-1}\text{s}^{-1},$$

which is orders of magnitude larger than that of copper, $-5.5\times10^{-11}\ \text{m}^3\text{A}^{-1}\text{s}^{-1}$ (refer to *Table 2.4*). As a matter of fact, this is why all Hall-effect devices use a semiconductor rather than a metal sample. (*1 pt*)