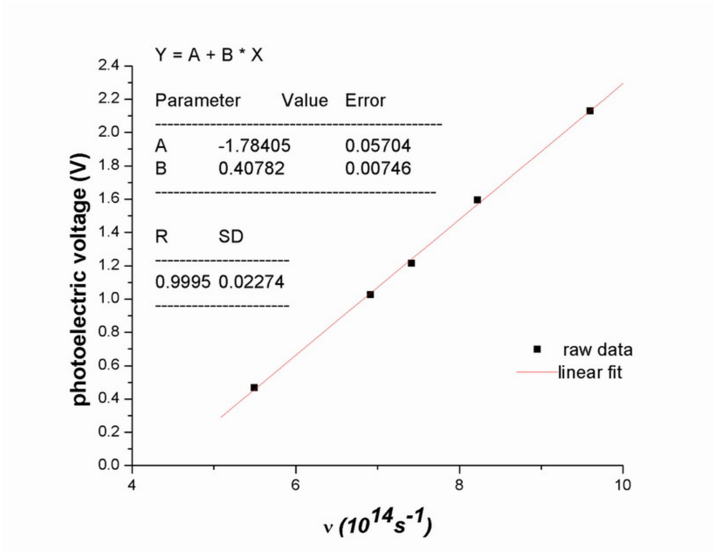
EMSE314--Fall 2003--Homework 3--Name: Changrong Li
Issued: Oct. 01, Due: Oct. 13

Problem 1—"photoelectric effect" (20 pts)



(a) $K.E. = h\nu - h\nu_0 = -e\varepsilon \Rightarrow -\varepsilon = \frac{h}{e}(\nu - \nu_0)$, where $\nu = c/\lambda$. Therefore, the "**$-\varepsilon$ vs. ν**" curve will yield a straight line. The slope is h/e, the intercept at ν axis $-h\nu_0/e$. Hence,

$$h = 1.6\times10^{-19}\times0.408\times10^{-14} = 6.53\times10^{-34}\,\text{Js},\quad \nu_0 = \frac{1.784\times1.6\times10^{-19}}{6.53\times10^{-34}} = 4.37\times10^{14}\,s^{-1},$$

work function $\phi = h\nu_0 = \frac{6.53\times10^{-34}\times4.37\times10^{14}}{1.6\times10^{-19}} = 1.78$ eV. (8 pts)

(b)

Electron States	E^B (eV)	$K.E. = h\nu - E^B - \phi$ (eV)
Conducting Electron	0	**1248.22**
2p	31	**1217.22**
2s	64	**1184.22**
1s	1070	**178.22**

The difference between kinetic energies reflects different binding energies of electrons in each orbit. The electrons that are closer to the nucleus are bound more tightly. Therefore, they show higher biding energies and lower kinetic energies. (4+2=6 pts)

(c) Suppose the light generated by UV lamp is monochromic, i.e. $\lambda = 254 nm$. Since the energy corresponding to this wavelength is much larger than the work function of sodium, we can assume each UV photon creates a single photoelectron. Therefore,

$$J = e\Gamma_{ph} = \frac{eI_{light}}{E_{ph}} = \frac{eI_{light}\lambda}{hc} = 40.9\,A/m^2.$$ (3 pts)

(d) The tungsten lamp at 1000K provides IR light with wavelength between 1000-7000 nm with max intensity at 3000 nm. Since the energy corresponding to this wavelength is lower than the work function of sodium, none of the photon has enough energy to create photoelectron. Therefore, the photoelectric current is zero. (3 pts)

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Problem 2--"particle in a box" (15 pts+5pts)

(a) Consider time-independent Schrödinger equation $\frac{d^2\psi(x)}{dx^2}+\frac{2m}{\hbar^2}(E-V)\psi(x)=0$ (Eq.1)

(i) If $x \geq L$ or $x \leq 0$, $\psi(x)=0$ since $V(x)=\infty$ but E can't be infinite. (1 pts)

(ii) If 0<x<L , V=0, $\psi(x)$ is determined by (1 pts) $\frac{d^2\psi(x)}{dx^2}+\frac{2mE}{\hbar^2}\psi(x)=0$. (Eq.2)

A general solution is (1 pts)

$$\psi(x)=A\exp(ikx)+B\exp(-ikx)=(A+B)\cos kx+i(A-B)\sin kx \quad \text{(Eq.3)}$$

The boundary condition is that $\psi(0)=0,\ \psi(L)=0$, i.e.

$$\begin{cases}\psi(0)=A+B=0\\ \psi(L)=(A+B)\cos(kL)-i(A-B)\sin(kL)=0\end{cases} \quad \text{(Eq.4)}$$

Since A and B can't both equal 0, we have

$$\begin{cases}\sin(kL)=0\\ A+B=0\end{cases} \quad \text{(Eq.5)}$$

Rewrite it, we get

$$\begin{cases}kL=n\pi \Rightarrow k_n=n\pi/L, \text{ where n is an integer;}\\ B=\text{-}A\end{cases} \quad \text{(2 pts)} \quad \text{(Eq.6)}$$

$$\psi_n(x)=i2A\sin(k_n x)=i2A\sin(n\pi x/L)$$

Since the total probability of finding the particle is one,

$$1=\int_0^L[\psi(x)\psi^*(x)]dx=\int_0^L 4A^2\sin^2(n\pi x/L)dx=2A^2\int_0^L[1-\cos(2n\pi x/L)]dx$$
$$=2A^2\left(x-\frac{\sin(2n\pi x/L)}{2n\pi/L}\right)\Big|_0^L=2A^2L\Rightarrow A=1/\sqrt{2L} \quad \text{(Eq.7)}$$

$$\psi_n(x)=i\sqrt{\frac{2}{L}}\sin(k_n x)=i\sqrt{\frac{2}{L}}\sin\frac{n\pi x}{L} \quad \text{(1 pts)} \quad \text{(Eq.8)}$$

Substituting this into Schrödinger equation,

$$E_n=\frac{\hbar^2 k_n^2}{2m}=\frac{n^2\hbar^2\pi^2}{2mL^2}=\frac{n^2h^2}{8mL^2} \quad \text{(1 pts)} \quad \text{(Eq.9)}$$

$$R_{p\to q}=\left|\int_0^L[\psi_p(x)\cdot(x-L/2)\cdot\psi_q(\mathrm{x})\ ]dx\right|=\frac{2}{L}\left|\int_0^L\sin(k_p x)\cdot(x-L/2)\cdot\sin(k_q x)\ dx\right|$$

(b) $$=\left|\frac{1}{L}\int_0^L(x-L/2)[\cos(k_{p-q}x)-\cos(k_{p+q}x)\ ]dx\right|$$

$$=\left|\frac{1}{L}\int_0^L x[\cos(k_{p-q}x)-\cos(k_{p+q}x)\ ]dx-\frac{1}{2}\int_0^L[\cos(k_{p-q}x)-\cos(k_{p+q}x)\ ]dx\right|$$

$$\int x(\cos tx)dx=\frac{1}{t}\int xd(\sin tx)=\frac{1}{t}[x\sin tx-\int(\sin tx)dx]=\frac{1}{t}x\sin tx+\frac{1}{t^2}\cos tx$$

$$\int\cos(tx)dx=\frac{1}{t}\sin(tx)$$

$$R_{p\to q}=\left|\frac{1}{L}\left(\frac{1}{k_{p-q}{}^2}\cos k_{p-q}x-\frac{1}{k_{p+q}{}^2}\cos k_{p+q}x\right)\Big|_0^L\right|$$

$$\begin{cases}=0,\ \text{if p-q is even (so that p+q=p-q+2q is also even);}\\ =\dfrac{2L}{\pi^2}\left[\dfrac{1}{(p-q)^2}-\dfrac{1}{(p+q)^2}\right],\ \text{if p-q is odd.}\end{cases}$$

Therefore, transitions $1\to 2$ is possible, but $1\to 3$ is impossible (8 pts)

(c) A selection rule is given by *p-q=odd*. (5 pts)

Problem 3--"Heisenberg uncertainty principle"

$$\Delta t\Delta E\cong\hbar\Rightarrow\Delta E=\frac{\hbar}{\Delta t}$$

$$\Delta\upsilon=\frac{\Delta E}{h}=\frac{1}{2\pi\Delta t}$$

$$\lambda=\frac{c}{\nu}\Rightarrow\frac{d\lambda}{d\nu}=-\frac{c}{\nu^2}=-\frac{\lambda^2}{c}$$

$$\Rightarrow\Delta\lambda=\frac{\lambda^2}{c}\Delta\nu=\frac{\lambda^2}{c}\frac{1}{2\pi\Delta t}=\frac{(589\times10^{-9})^2}{3\times10^8\times2\times3.1416\times20\times10^{-9}}m=9.20\times10^{-15}m$$ (5 pts)

Problem 4--"hydrogenic atom" (10 pts)

(b) Ψ_3 is the only one with spherical symmetry (independent of θ and ϕ), so it must be 2s.
Ψ_1 is independent of ϕ, i.e. it is independent to x and y, so it must be $2p_z$.
When y=0, cos ϕ =0 and Ψ_2=0. Therefore, Ψ_2 must be $2p_x$.
When x=0, sin ϕ =0 and Ψ_4=0. Therefore, Ψ_4 must be $2p_y$. (4 pts)

(a) (6 pts)

Ψ	*Ψ₁*	*Ψ₂*	*Ψ₃*	*Ψ₄*
State	$2p_z$	$2p_x$	*2s*	$2p_y$
l	**1**	**1**	**0**	**1**
m	**0**	**1**	**0**	**-1**