

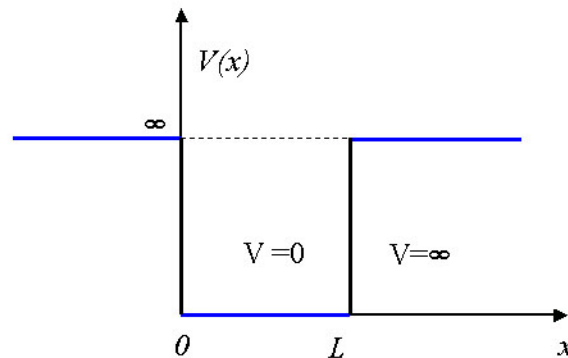
(4 Problems, 50pts +5 bonus pts)

Problem 1—“photoelectric effect” (20 pts)

The incident light wavelength vs. photoelectric voltage on sodium metal is given as the following: (after R.A. Millikan, Phys. Rev., **7** (1916), 355)

λ , nm	312.5	365.0	404.7	433.9	546.1
$-\epsilon$, volt	2.128	1.595	1.215	1.025	0.467

- (a) Calculate the work function of sodium in eV and the Planck constant. (6 pts)
- (b) If the sodium surface is bombarded by a Mg K_α X-ray ($h\nu = 1250$ eV), calculate largest possible kinetic energy of the photoemitted electrons in eV. Given the **binding energy** of Na 1s, 2s, and 2p states are 1070eV, 64eV, and 31 eV, respectively; calculate the kinetic energy of electron emitted from these states. Why are they different? (8 pts)
- (c) Suppose the sodium surface is incident by UV lamp with wavelength between 150-370nm and a maximum density of 20 mW/cm² at 254nm, what will be the maximum photoelectric current density? (3 pts)
- (d) Suppose the sodium surface is incident by a tungsten lamp (Refer to *Figure 3.11* in the textbook) working at 2500K with an intensity of 20 mW/cm², what will be the maximum photoelectric current density? (3 pts)
- Note: Suppose the light generated by UV lamp and W lamp are both monochromic, which adopts the maximum intensity (In real world, this is not a good assumption).*

Problem 2--“particle in a box” (15 pts+5pts)

- (a) For a particle in a potential well as shown in the diagram, derive the wave function and energy by solving the Schrödinger equation. (7 pts)
- (b) For an electron at the center of potential well, determine if transitions $1 \rightarrow 2$ and $1 \rightarrow 3$ are possible. (8 pts)

Hint: The probability of transition $p \rightarrow q$ is proportional to $R_{p \rightarrow q} = \left| \int_0^L [\psi_p \cdot (x - \frac{L}{2}) \cdot \psi_q] dx \right|$

where $\Psi_p(x)$ and $\Psi_q(x)$ are wave functions for integer $n=p$ and $n=q$, respectively.

- (c) Guess the selection rule for the particle in a box. (+5 points)

Problem 3—“Heisenberg uncertainty principle” (5 pts)

An excited electron in a sodium atom emits a radiation of wavelength 589 nm and returns to the ground state. If the mean time for the transition is about 20 ns, calculate $\Delta\lambda$, the inherent width of the emission in the sense of wavelength.

Problem 4--“hydrogenic atom” (10 pts)

The solution to Schrödinger equation of hydrogenic-atom when $n=2$ is given as

$$\psi_1 = \frac{1}{4\sqrt{2}\pi} \left(\frac{Z}{a_0} \right)^{\frac{3}{2}} \rho e^{-\frac{\rho}{2}} \cos\theta$$

$$\psi_2 = \frac{1}{4\sqrt{2}\pi} \left(\frac{Z}{a_0} \right)^{\frac{3}{2}} \rho e^{-\frac{\rho}{2}} \sin\theta \cos\phi$$

$$\psi_3 = \frac{1}{4\sqrt{2}\pi} \left(\frac{Z}{a_0} \right)^{\frac{3}{2}} (2-\rho) e^{-\frac{\rho}{2}}$$

$$\psi_4 = \frac{1}{4\sqrt{2}\pi} \left(\frac{Z}{a_0} \right)^{\frac{3}{2}} \rho e^{-\frac{\rho}{2}} \sin\theta \sin\phi,$$

where $\rho = \frac{Zr}{a_0}$, Z is atomic number.

(a) Identify l and m for each function. Show your reason. (6 pts)

(b) Which wave function denotes 2s state? How about 2p_x, 2p_y, and 2p_z state? (4 pts)

Hint: consider the symmetry of each function.