

**Problem 1— “DOS for a 2D electron gas” (14 pts)**

Consider a two-dimensional electron gas in which the electrons are restricted to move freely within a square area  $a^2$  in the  $xy$  plane, i.e.  $0 \leq x \leq a$  &  $0 \leq y \leq a$ . Show that the density of states  $g(E)$  is constant (independent of energy).

*Hint: Following the procedure in Section 4.5 (in the textbook).*

**Problem 2— “Diamond and Tin” (14 pts)**

Germanium, silicon, and diamond have the same crystal structure, the diamond structure. Bonding in each case involves  $sp^3$  hybridization. The bonding energy decreases as we go from C to Si to Ge, as noted in following table:

Property	Diamond	Silicon	Germanium	Tin
Melting temperature, $^{\circ}\text{C}$	3800	1417	937	232
Covalent radius, nm	0.077	0.117	0.122	0.146
Bond energy, eV	3.60	1.84	1.7	1.2
First ionization energy, eV	11.26	8.15	7.88	7.33
Bandgap, eV	?	1.12	0.67	?

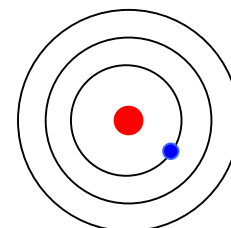
a). What would you expect for the band gap of diamond? How does it compare with the experimental value of 5.5 eV?

b). Tin has a tetragonal crystal structure, which makes it different than its group members, diamond, silicon, and germanium. (i). Is it a metal or a semiconductor? (ii). What experiments do you think would expose its semiconductor properties?

*Hint: Make four individual plots of the band gap versus each property.*

**Problem 3—“Bohr model of hydrogen atom” (10 pts)**

Bohr’s “planetary” model of the universe assumes that electrons can move in stable circular orbits around a nucleus. It is assumed that each orbit must correspond to an integer number of wavelengths:  $2\pi r_n = n\lambda$ . For a single electron of mass  $m$  in a circular orbit of radius  $r$  about a fixed nucleus of charge  $Ze$ , the Coulomb and centripetal forces must balance.



a) Derive expressions for the orbit radius  $r_n$ , velocity  $v_n$ , and the binding energy  $E_n$  for hydrogen atom, as a function of  $n$ , the index of the orbit.

b) Find the numerical values for  $r_1$  in nm,  $v_1$  in m/s, and  $E_1$  in eV.

*Hint: Employing the de Broglie relationship.*

**Problem 4 Fermi energy and electron concentration (12 pts)**

Consider the metals from groups I, II and III in the following table. Calculate the Fermi energies at 0K, and compare the values with the experimental values.

Metal	Group	$M_{at}$ (g/mol)	Density ( $\text{g cm}^{-3}$ )	$E_F$ (eV)[Calculated]	$E_F$ (eV)[Experiment]
Cu	I	63.55	8.96	?	6.5
Zn	II	65.38	7.14	?	11.0
Al	III	26.98	2.70	?	11.8