EMSE314--Fall 2003--Homework 5--Name: Changrong LI
Issued: Nov. 03, Due: Nov. 10

Problem 1—"Metal-metal contact" (10 pts)

a). The quantum leak, or tunneling through an energy barrier, is responsible. (3 pts)
b). Following the discussion in Section 3.5 of the textbook, the probability of tunneling through an energy barrier V_o, i.e. the transmission coefficient T is given by

$$T = \frac{1}{1 + D\sinh^2(a\alpha)}, \qquad \text{(Eqn. 1)}$$

where $D = \frac{V_0^2}{4E(V_0 - E)}$, (Eqn. 2)

and $\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}}$. (Eqn. 3)

Consider V_0~2.5eV, E~1eV, a~5nm. We can calculate that

$$\alpha = \sqrt{\frac{2m(V_0 - E)}{\hbar^2}} = \sqrt{\frac{2\times 9.1\times 10^{-31}\times(2.5-1)\times 1.6\times 10^{-19}}{(1.05\times 10^{-34})^2}} = 6.3\times 10^9\,\text{m}^{-1},$$

and $a\alpha = 6.3\times 10^9 \times 5\times 10^{-9} = 31.5$. Since it is much greater than unity, Eqn.(1) is reduced to $T = T_0 \exp(-2a\alpha)$ (Eqn. 4)

where $T_0 = \frac{16E(V_0 - E)}{V_0^2}$. (3 pts) (Eqn. 5)

i) Consider the same energy barrier V_0=2.5eV and the same E=1eV.
$\alpha = 6.3\times 10^9\,\text{m}^{-1}$, and $T_0 = \frac{16E(V_0 - E)}{V_0^2} = \frac{16\times 1\times(2.5-1)}{2.5^2} = 3.84$.

When a=5nm, $a\alpha = 31.5$, $T = 3.84\times\exp(-2\times 31.5) = 1.67\times 10^{-27}$;

When a=1nm, $a\alpha = 6.3$, $T = 3.84\times\exp(-2\times 6.3) = 1.3\times 10^{-5}$.

ii) Consider the same energy barrier V_0=2.5eV, and width a=5nm.

When E=1eV, $\alpha = 6.3\times 10^9\,\text{m}^{-1}$, $T_0 = 3.84$, $T = 3.84\times\exp(-2\times 6.3\times 5) = 1.67\times 10^{-27}$;

When E=2eV, $\alpha = 3.63\times 10^9\,\text{m}^{-1}$, $T_0 = 2.56$, $T = 2.56\times\exp(-2\times 3.63\times 5) = 4.4\times 10^{-16}$.

We can easily see how important the applied voltage and the thickness of the oxide in this phenomenon are. (4 pts)

Problem 2— "temperature dependence of conductivity" (15 pts)

a) Since $N_d \gg n_i \Rightarrow n = N_d$ & $p = n_i^2/n = n_i^2/N_d \ll n$.

The conductivity at room temperature T=300K is

$$\sigma = eN_d\mu_e = 1.6\times 10^{-19}\times(10^{15}\times 10^6)\times(1350\times 10^{-4}) = 21.6\Omega^{-1}\text{m}^{-1} \text{ (2 pts)}$$

b) At T=T_i, the intrinsic concentration $n_i = N_d = 10^{15}$ cm^{-3}. From Figure 5.16 (page 346), the graph n_i(T) vs. 1/T, we have $1000/T_i = 1.8 \Rightarrow T_i = 1000/1.8 = 556$ K or 283°C. (2 pts)

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c) The ionization region ends at T=T_s, when all donors have been ionized. At T=T_s,

$$n = N_d = (\frac{1}{2}N_c N_d)^{1/2}\exp(-\frac{\Delta E}{2kT_s}),$$

$$\therefore\ T_s = \frac{-\Delta E}{2k\ln\sqrt{2N_d/N_c}} = \frac{\Delta E}{k\ln\frac{N_c}{2N_d}}.$$

$N_d=10^{15}$ cm^{-3}, $\Delta E = 0.045$ eV, assuming $N_c = 2.8\times10^{19}$ cm^{-3} @$300K$,

$$\therefore\ T_s^0 = \frac{\Delta E}{k\ln\frac{N_c}{2N_d}} = \frac{0.045\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln\frac{2.8\times10^{19}}{2\times10^{15}}} = 54.65K.$$ (3 pts)

Finding the new N_c at this temperature,

$$N_c^1 = 2(\frac{2\pi m_e^* k}{h^2}T)^{3/2} = 2.8\times10^{19}\times(\frac{54.65}{300})^{1.5} = 2.179\times10^{18}\text{cm}^{-3},$$

$$\therefore\ T_s^1 = \frac{\Delta E}{k\ln\frac{N_c^1}{2N_d}} = \frac{0.045\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln\frac{2.179\times10^{18}}{2\times10^{15}}} = 74.64K.$$ (2 pts)

We can go further.

$$N_c^2 = 2(\frac{2\pi m_e^* k}{h^2}T)^{3/2} = 2.8\times10^{19}\times(\frac{74.64}{300})^{1.5} = 3.475\times10^{18}\text{cm}^{-3},$$

$$\therefore\ T_s^2 = \frac{\Delta E}{k\ln\frac{N_c^2}{2N_d}} = \frac{0.045\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln\frac{3.475\times10^{18}}{2\times10^{15}}} = 69.97K.$$

$$N_c^3 = 2(\frac{2\pi m_e^* k}{h^2}T)^{3/2} = 2.8\times10^{19}\times(\frac{69.97}{300})^{1.5} = 3.154\times10^{18}\text{cm}^{-3},$$

$$\therefore\ T_s^3 == \frac{\Delta E}{k\ln\frac{N_c^3}{2N_d}} = \frac{0.045\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln\frac{3.154\times10^{18}}{2\times10^{15}}} = 70.89K.$$

$$N_c^4 = 2(\frac{2\pi m_e^* k}{h^2}T)^{3/2} = 2.8\times10^{19}\times(\frac{70.89}{300})^{1.5} = 3.216\times10^{18}\text{cm}^{-3},$$

$$\therefore\ T_s^4 = \frac{\Delta E}{k\ln\frac{N_c^3}{2N_d}} = \frac{0.045\times1.6\times10^{-19}}{1.38\times10^{-23}\times\ln\frac{3.216\times10^{18}}{2\times10^{15}}} = 70.67K$$

We can conclude that T_s =70.7 K.

d) Refer to Figures 5.15 and 5.14. (4 pts)

e) Refer to Figure 5.20. (2 pts)

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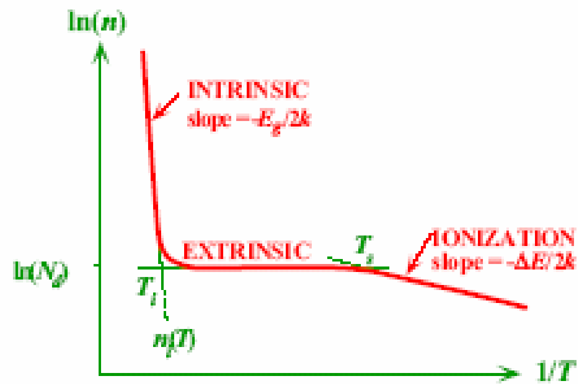

Fig. 5.15: The temperature dependence of the electron concentration in an n-type semiconductor.

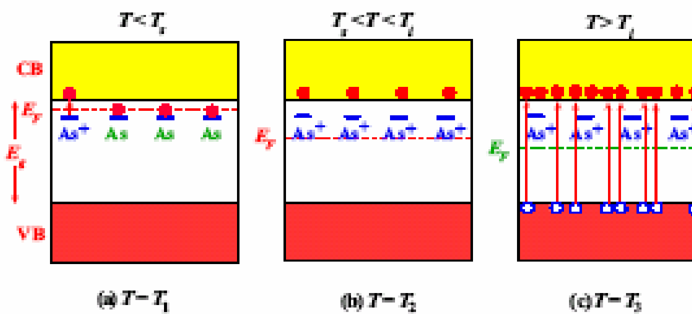

Fig. 5.14: (a) Below T_S, the electron concentration is controlled by the ionization of the donors. (b) Between T_S and T_i, the electron concentration is equal to the concentration of donors since they would all have ionized. (c) At high temperatures, thermally generated electrons from the VB exceed the number of electrons from ionized donors and the semiconductor behaves as if intrinsic.

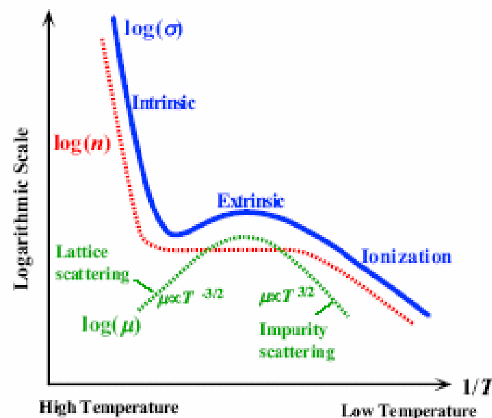

Fig. 5.20: Temperature dependence of electrical conductivity for a doped (n-type) semiconductor.

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Problem 3—"Schottky & ohmic contact" (13 pts)

Since $N_d(=10^{16}\text{cm}^{-3}) \gg n_i(=1.45\times10^{10}\text{cm}^{-3})$, we have $n = N_d = 10^{16}\text{cm}^{-3}$.

For intrinsic Si, $n_i = N_c \exp\left[-\dfrac{E_c - E_{Fi}}{kT}\right]$,

while for *n*-type Si, $n = N_c \exp\left[-\dfrac{E_c - E_{Fn}}{kT}\right] = N_d$

$$\therefore \quad \frac{N_d}{n_i} = \exp\left[-\frac{E_{Fn} - E_{Fi}}{kT}\right] \Rightarrow E_{Fn} - E_{Fi} = kT\ln\left(\frac{N_d}{n_i}\right) = 0.0259\ln\left(\frac{10^{16}}{1.45\times10^{15}}\right) = 0.348\text{eV}$$

Moreover, $E_{Fi} = E_v + \dfrac{1}{2}E_g - \dfrac{3}{4}kT\ln\left(\dfrac{m_e^*}{m_h^*}\right) = E_c - \dfrac{1}{2}E_g - \dfrac{3}{4}kT\ln\left(\dfrac{m_e^*}{m_h^*}\right)$,

$$\therefore \quad E_c - E_{Fi} = \frac{1}{2}E_g + \frac{3}{4}kT\ln\left(\frac{m_e^*}{m_h^*}\right) = (0.553 \pm 0.010)\text{eV}$$

$$=(\frac{1}{2}\times1.10 + 0.75\times0.0259\times\ln\frac{0.26}{0.38} = 0.543\text{eV}) \text{ or } (\frac{1}{2}\times1.10 + 0.75\times0.0259\times\ln\frac{1.08}{0.56} = 0.563\text{eV})$$

Hence, $\Phi_n = \chi + E_c - E_{Fn} = \chi + (E_c - E_{Fi}) - (E_{Fn} - E_{Fi})$

$=4.01+(0.553\pm0.010)-0.348 = (4.125\pm0.010)$ eV. (3 pts)

a) For a Schottky contact, $\Phi_m > \Phi_n$. Possible metals are Al and Au. (2 pts)

b) For an Ohmic contact, $\Phi_m < \Phi_n$. Possible metals are Cs and Li. (2 pts)

c) In the case that both B and C are ohmic contacts, the current is not limited by the contacts, but by the resistance R of the Si outside the contact region. In other words, I=V/R, as shown in Figure *HW 5-3c*.Moreover, $\sigma = eN_d\mu_e = 216\Omega^{-1}\text{m}^{-1}$,

$$R = \frac{L}{\sigma A} = \frac{10^{-4}}{216\times10^{-5}\times10^{-5}} = 4.63\times10^{-3}\Omega$$. (2 pts)

d). In the case that B is ohmic and C is a Schottky junction, the I-V curve is the same as a regular Schottky junction. In other words, $J = J_0\left[\exp\left(\dfrac{eV}{kT}\right) - 1\right]$, as shown in Figure *HW 5-3d*. (2 pts)

e). In the case that both B and C are Schottky contacts, $J = \pm J_0\left[\exp\left(\dfrac{\mp eV}{kT}\right) - 1\right]$.

as shown in Figure *HW 5-3e*. (2 pts)

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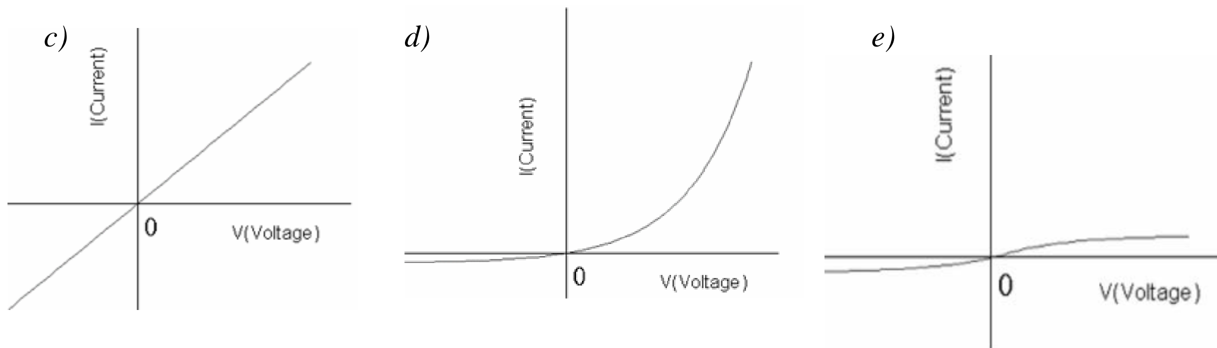

Figure *HW 5-3d*

Problem 4—"thermal drift in semiconductor devices" (12 pts)

a). In a *p*-type semiconductor (SC), we can assume that we only have holes as the mobile charge carriers. The acceptors are negatively charged. The temperature gradient results in the diffusion of holes (instead of electrons) from the hot to cold end. This exposes negative acceptors in the hot region (instead of positive charge as in the *n*-type SC). *Thus in a p-type semiconductor, the Seebeck effect has the reverse sign, or the polarity of the Seebeck voltage is reversed with respect to that for an n-type SC for the same temperature gradient.* (3 pts)

b) It is clear that S_n depends on (E_c-E_F), as shown in the following table.

N_d (cm^{-3})	(E_c-E_F) (eV)	S_n(mV/K)
10^{14}	0.3244	-1.25
10^{16}	0.2053	-0.86
10^{18}	0.0862	-0.46

$$N_d = N_c \exp\left[-\frac{E_c - E_{Fn}}{kT}\right] \Rightarrow E_c - E_{Fn} = -kT\ln\left(\frac{N_d}{N_c}\right).$$

For example,

$$N_d = 10^{14}\,\text{cm}^{-3},\ E_c - E_{Fn} = -kT\ln\left(\frac{N_d}{N_c}\right) = -\frac{1.38\times10^{-23}\times300}{1.6\times10^{-19}}\times\ln\left(\frac{10^{14}}{2.8\times10^{19}}\right) = 0.3244\text{eV}$$

$$S_n = -\frac{k}{e}\left[2 + \frac{(E_c - E_F)}{kT}\right] = -\frac{1.38\times10^{-23}}{1.6\times10^{-19}}\left[2 + \frac{0.3244\times1.6\times10^{-19}}{1.38\times10^{-23}\times300}\right] = -1.25\text{mV/K}$$ (8 pts)

In a typical semiconductor device such as an IC there will be many junctions between differently doped materials. A small local temperature fluctuation can give rise to a temperature induced signal of the order of few millivolts. In a dc op amp of large gain this will appear as a spurious signal in the output. One design cure is to use two different input devices at similar temperatures and use the output signal difference between them. (1 pts)